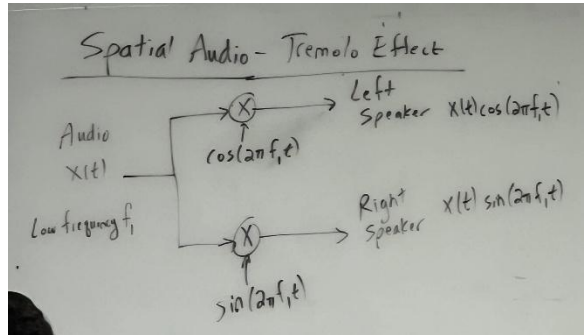


Lecture 5 on Sampling & Aliasing Part 3

Tuesday, September 22, 2026

By TA Alice Liu

Demonstration of audio effects

- **Tremolo.** The Smiths, "[How Soon is Now?](#)", opening guitar riff has a tremolo effect by guitarist Johnny Marr, with a second voice playing slide guitar. Tremolo is rapid change in volume to give a sensation of motion, and is rapidly varying amplitude modulation effect. "[Songs that Changed Music: The Smiths - How Soon Is Now](#)".
- **Another Tremolo Effect: Spatial Audio.** Play an audio signal that ping-pongs between the left and right audio channels (speakers) using amplitude modulation. When cosine is 1, the sine is zero, and vice-versa. The cosine and sine signals have low frequencies such as 1 Hz or 5 Hz.
 
- **Vibrato.** Chris Isaak, "[Wicked Games](#)", an example of vibrato effect using a guitar whammy bar. Vibrato is a slowly varying frequency modulation effect.
- **Delay.** Stella, [Reverb and Delay Explained](#), start at 1:11 mark. Delay controls when echoes arrive, whereas reverb consists of delays that are not controlled.

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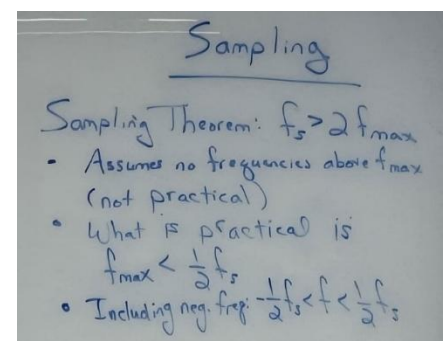
Logistics:

- Mini-Project #1 Due Wednesday by 11:59pm on the course Canvas page
- Homework #3 to be assigned, due Friday by 11:59pm

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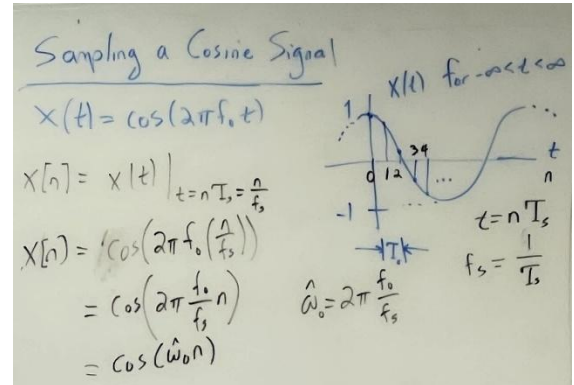
Review of Sampling

The Sampling Theorem says that if a continuous-time signal $x(t)$ has no frequencies higher than $f_{\max}$, then the signal can be reconstructed from its samples if the sampling rate c is chosen to be $f_s > 2 f_{\max}$. This is a wonderful theoretical result upon which practice builds on. When sampling at sampling rate f_s , frequencies up to but not including $\frac{1}{2} f_s$ are captured.



Sampling and Reconstruction

- Continuous Time discussion
 - o To properly recreate the signal (all its frequency components), sampling frequency, f_s , must satisfy: $f_s > 2 * f_{max}$, where f_{max} is the highest frequency component in the signal
 - o Alternatively: given a sampling frequency, f_s , we can reconstruct any signal where its highest frequency component satisfies: $f_{max} < \frac{1}{2} f_s$
 - o Otherwise, we will have **aliasing**
- Discrete Time discussion
 - o Given a continuous time sampling frequency, f_s , in cycles per **second**, and a continuous time signal with a maximum frequency component, f_0 , in cycles per **second**, the discrete time sampling frequency, $\hat{\omega}_0$, that satisfies the Sampling Theorem should be: $\hat{\omega}_0 = 2\pi \frac{f_0}{f_s}$, in radians per **sample**, where $-\pi \leq \hat{\omega}_0 \leq \pi$
 - o **Else, if the Sampling Theorem does NOT hold, then ALIASING occurs, and $\hat{\omega}_0$ must be adjusted!**
- Aliasing Example (see Slides 5-9)



If $f_s > 2f_0$,
 $\hat{\omega}_0 = 2\pi \frac{f_0}{f_s}$ in rad/sample
 is unchanged: $-\pi < \hat{\omega}_0 < \pi$

Otherwise, we have **aliasing**.
 $\hat{\omega}_0$ will be reduced
 by offset $2\pi m$ for
 integer m until $-\pi < \hat{\omega}_0^{new} < \pi$

Slide 5-9

Slide 5-9

Let $f_0 = f_0^{new} + 2f_s$

What about $2\pi \frac{f_0}{f_s} = \hat{\omega}_0 = 4.4\pi$?

Aliases to 0.4π .

$2\pi \frac{f_0^{new} + 2f_s}{f_s} = 2\pi \frac{f_0^{new}}{f_s} + 2\pi \frac{2f_s}{f_s}$

$2\pi \frac{f_0^{new}}{f_s} = \hat{\omega}_0^{new} = 0.4\pi$

Aliasing

1. Classic "Aliasing"
 - a. Cause: Sampling rate f_s does not satisfy the Sampling Theorem
 - b. Results: After sampling, consider a frequency f_0 that was captured correctly by sampling because $f_s > 2f_0$. However, there are also an infinite number of

frequencies outside the range $-\frac{1}{2} f_s < f < \frac{1}{2} f_s$ that will alias to frequency f_0 due to sampling. These aliasing frequencies are of the form $f_0 + m f_s$ where m is an integer.

2. Another kind of aliasing: Folding

- a. Cause: Sampling rate f_s does not satisfy the Sampling Theorem
- b. Results: After sampling, consider a frequency f_0 that was captured correctly by sampling because $f_s > 2 f_0$. However, there are also an infinite number of frequencies outside the range $-\frac{1}{2} f_s < f < \frac{1}{2} f_s$ that will alias to frequency f_0 due to sampling. These aliasing frequencies are of the form $-f_0 + m f_s$ where m is an integer.

Reconstruction

- Ideal Reconstruction
 - o Sampling satisfies Sampling Theorem
 - o Not realistic
 - We can't sample every possible point
 - o Not practical
 - We don't **need** every possible point, we can work off of fewer samples
 - Saves space, compute
- Interpolation between sampled points
 - o Zero-order Hold
 - Pro: Simple implementation
 - Con: clear artifacts
 - Application: used in some designs for digital-to-analog converters
 - o Linear
 - Pro: Pretty good approximation
 - Con: more work, still has artifacts
 - Application: used in speech and audio signals; causes blurring effects in images
 - o Curve-fitting
 - Pro: smooth curve
 - Con: complex implementation
 - Application: commonly used when expanding an image and interpolating the pixel values for the new pixels (e.g. expanding a 512 x 512 image into a 2048 x 2048 image).